STRESS CONCENTRATION IN GLASS-EPOXY COMPOSITE PLATES

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THESIS

STRESS CONCENTRATION
IN
GLASS-EPOXY COMPOSITE PLATES

by

Sakol Vudhivai

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The results are reported in the form of the stress concentration factor, K_{tg}, as a function of two nondimensional parameters, one representing the influence of the

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Stress Concentration
in
Glass-Epoxy Composite Plates

by

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requirements for the degree of

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TABLE OF SYMBOLS AND ABBREVIATIONS

Ag	Gross cross-sectional area
a	Hole diameter
c	Distance from the center of the hole to the near edge of the plate
e	Eccentricity, distance from the center of the hole to the far edge of the plate
E	Young's Modulus
K_{tg}	Stress concentration factor
P	Load applied by the tension machine
σ	Average stress
σ_H	Stress at the edge of the hole
t	Thickness of the plate or specimen
b	Width of the plate or specimen
ε	Strain
ω	Uncertainty value associated with a particular measurement
in.	Inch
lbs.	Pound-force
psi	Pound-force per square inch

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The material was fabricated in the Aeronautics Department shop under the supervision of Professor Milton H. Bank.

The apparatus used to manufacture the specimen, for example, the heaters and relays, was prepared in the Mechanical Engineering Department shop.

The preparation of specimens, consisting of machining operations and installation of gages, was done in the Mechanical Engineering Department shop.

The research work was performed in the Aeronautics Department Laboratory.

It has been evident, during the period of two quarters since the beginning of this research, as a constant, the cooperative help received from Mr. Kenneth Mothersell and the technicians of the Mechanical Engineering Department shop.

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I. INTRODUCTION

The word "composite" in composite material signifies that two or more materials are combined on a macroscopic scale to form a useful material. The advantage of composites is that they usually exhibit the best qualities of their constituents and often some qualities that neither constituent possesses. The properties that can be improved by forming a composite material include:

1. Strength
2. Stiffness
3. Corrosion resistance
4. Wear resistance
5. Attractiveness
6. Weight
7. Fatigue life
8. Temperature-dependent behavior
9. Thermal insulation
10. Thermal conductivity
11. Acoustical insulation

Composite materials have a long history of usage. Their beginnings are unknown, but all recorded history contains references to some form of composite material. For example, straw was used by the Israelites to strengthen mud bricks. Plywood was used by the ancient Egyptions when they realized that would could be rearranged to achieve superior strength and resistance to thermal expansion as well as to swelling

owing to the presence of moisture. Medieval swords and armor were constructed with layers of different materials. More recently, fiber-reinforced resin composites that have high strength-to-weight and stiffness-to-weight ratios have become important in weight-sensitive applications such as aircraft and space vehicles.

There are three common types of composite materials:

1. Fibrous composites which consist of fibers in a matrix.
2. Laminated composites which consist of layers of various material.
3. Particulate composites which are composed of particles in a matrix.

Composites can be made that have the same strength and stiffness as high-strength steel, yet are 70 percent lighter [Ref. 2]. Other advanced composites are as much as three times as strong as aluminum, the common aircraft structural material, yet weigh only 60 percent as much. Moreover, as has already been noted, composite materials can be tailored to efficiently meet design requirements of strength, stiffness, and other parameters in various directions. These advantages should lead to new aircraft and space designs that are radical departures from past efforts based on conventional materials.

The impact of composite materials used on jet engine performance is also very substantial. Currently, with various metal alloys, a thrust-to-weight ratio of five to

one is achieved. Ultimately, with advanced graphite fiber composites, thrust-to-weight ratios on the order of 40 to one appear possible [Ref. 2].

In the near future, aircraft will be built with a very high percentage of components made from composite materials.

Lack of confidence is recognized as a primary inhibiting factor in a widespread use of advanced composites [Ref. 6]. Confidence in the performance and reliability of composite structures is, of course, the primary requirement. This lack of information constituted a motivation for the study in this thesis. In order to expand the data base on composite materials, it was decided to study the problem of stress concentration around holes out of center in thin plates with finite width.

II. BACKGROUND

When a hole is made in a plate, the stress field in the vicinity of the hole is altered, and stress concentrations result. It is unfortunate but true that such holes must be cut for bolts and fasteners, access panels and windows, and the like. So engineers must learn to analyze the stress concentrations which result from such holes and to make allowances for them in design.

The first solution of the fundamental problem of a finite circular hole in an infinite plate of isotropic material was made by Kirsch in 1898 [Ref. 7]. Solutions for holes near the edges of semi-infinite and finite width plates followed. Compilations of these solutions are to be found in books by Peterson [Ref. 3] and Savin [Ref. 8].

The problem of stress concentrations around a circular hole out of center in a plate of finite width made of isotropic material has been solved by S. Sjostrom [Ref. 9]. The same problem for a non-isotropic material, such as a laminated composite, is much more difficult, and no closed form solution is available. This problem has been treated in two ways: first, using the linear theory of anisotropic elasticity as discussed by Lekhnitskii [Ref. 10] and Savin [Ref. 8] and, second, using the finite element method [Ref. 12].

An initial study of this problem was reported by Alves [Ref. 5]. His work was limited by the size of his

specimens, and he was forced to use an elaborate extrapolation procedure to deduce the stress concentrations at the edge of the hole. It was decided to perform experiments similar to those done by Alves but this time using larger specimens which would permit correct measurement of the strain at the hole. Thus Alves' data and conclusions could be verified, and additional data could be generated which would add to the confidence in composite material applications.

III. SCOPE OF THE TEST PROGRAM

The objective of this thesis work was to take data which would permit drawing a diagram of the stress concentration factor K_{tg} at the edge of the hole at a position 90° from the longitudinal axis as a function of two nondimensional parameters involving:

1. Influence of hole size (Refer to column a/c in Table I).
2. Influence of eccentricity of the hole (Refer to column e/c or f in Table I).

Alves [Ref. 5] experienced difficulty in his tests of plates with holes because his specimens were too thin and too small. This lack of thickness forced him to attach his strain gage to the plane surface of the plate at a small distance from the hole, and thus he had to extrapolate to the hole from his data. The small size of his specimens caused problems in producing a uniform stress field in the test area.

Alves recommended that future specimens have a size sufficient to provide more uniform stress field away from the hole and the grip tabs. Also thicker specimens would allow installation of gages on the inner surface of the hole.

This thesis continued Commander Alves' work, this time using much larger and thicker specimens, so that strain gages could be applied inside the hole.

The objective of the work was to produce a plot of stress concentration factor versus hole location and hole size in finite composite tension glass epoxy specimens.

It was decided to restrict the experiments to only crossply lay-up (0°/90°) since time was limited, as one quarter was spent on designing the machine to manufacture the specimens. Also it was decided to reuse the same plate of laminate for subsequent experiments by enlarging the hole's size at a specific eccentricity after each test. Thus three individual specimens were manufactured, each with a different hole eccentricity, and each was tested four times with different hole sizes. This procedure presented as a by-product the advantage of minimizing variations in results caused by material inconsistencies, since all specimens representing a particular type of laminate were cut from the same laminated plate.

It was also decided that a total of twelve tests would be a reasonable number to provide a minimum amount of data necessary to permit the plotting of desired curves.

For this test creep was not taken into account. All the data to be obtained were intended to be valid for short-term loading and relatively low loading rates.

Two sizes of strain gages were used: very small gages (.031 x .032 in.) which were installed on the inside edge of the hole to measure the strain directly, and much larger gages which were used to check the uniformity of stress across the plate (See Figure 1).

IV. DESCRIPTION OF THE SPECIMENS

The specimens used in this testing program had nominal dimensions of six inches by 36 inches, with the thickness of .077 inch, and were made of glass-reinforced plastic epoxy laminate, with one circular hole displaced from the center, as shown in Figures 1 and 2.

The representative group of specimens had a programed distribution of parameters, as shown in Table I.

The specimens were crossply, having nine plies oriented 90° to each other alternately.

Three-inch wide end tabs, made of glass-cloth (see Figure 1) and epoxy (reinforced plastic) surfaced with thin copper plate, were glued to each end of the specimens, as indicated in Figure 1.

These tabs were attached to the whiffle tree fixture by four equally spaced bolts (See Figure 3 A and B) to spread the load as uniformly as possible over the specimen ends.

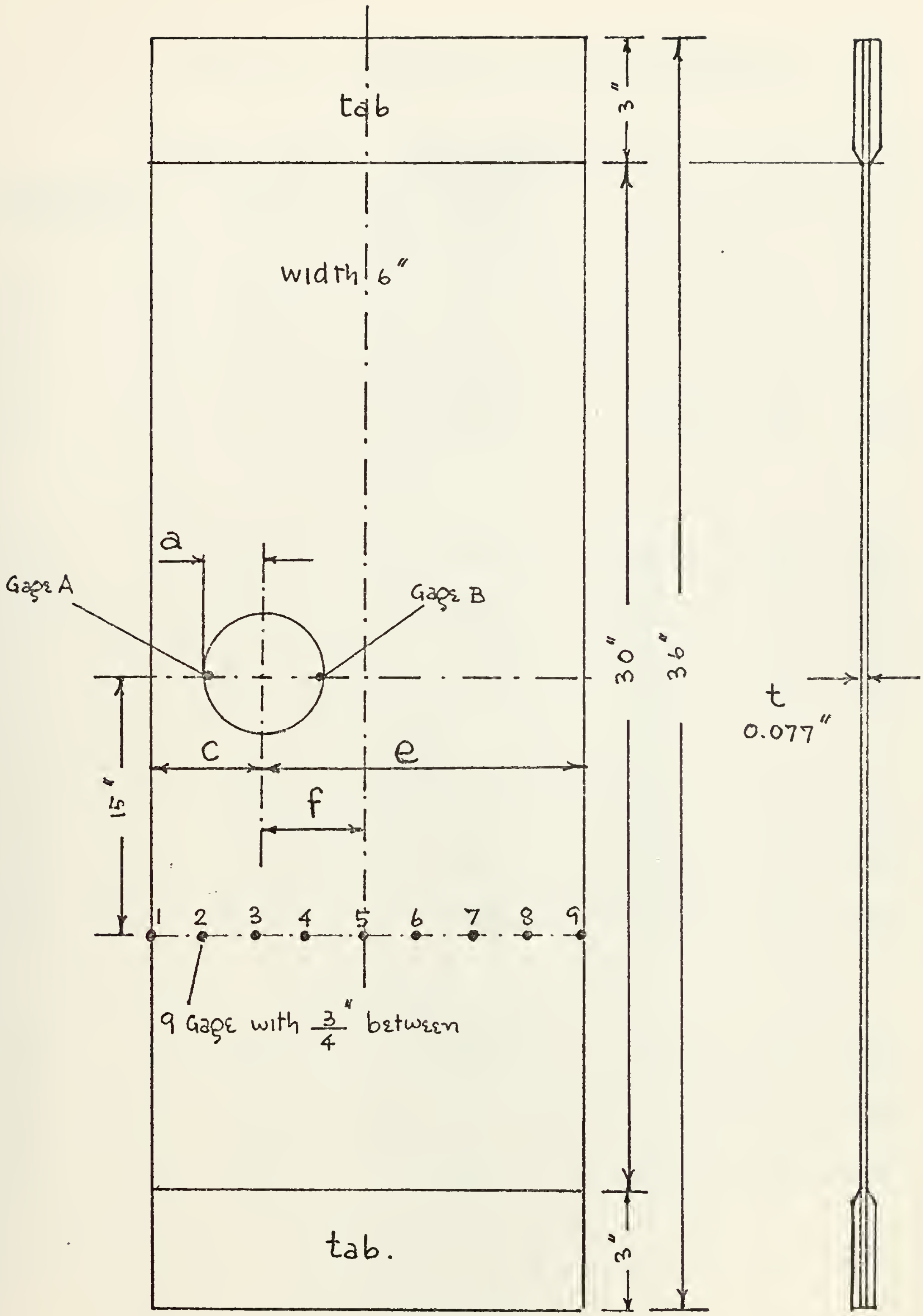

Figure 1. Basic Stress Concentration Specimen Dimension with the Position of Strain Gage Installation.

TABLE I. DIMENSIONS OF THE SPECIMENS

Specimen Number	Hole Radius (a) inch	Hole Center Location (f) inch	Hole Location Parameter c	e	a/c	e/c
A1	0.25	1.00	2.00	4.00	.125	2.00
A2	0.25	1.50	1.50	4.50	.167	3.00
A3	0.25	2.00	1.00	5.00	.250	5.00
B1	0.50	1.00	2.00	4.00	.250	2.00
B2	0.50	1.50	1.50	4.50	.333	3.00
B3	0.50	2.00	1.00	5.00	.500	5.00
C1	0.75	1.00	2.00	4.00	.375	2.00
C2	0.75	1.50	1.50	4.50	.500	3.00
C3	0.75	2.00	1.00	5.00	.750	5.00
D1	1.00	1.00	2.00	4.00	.500	2.00
D2	1.00	1.50	1.50	4.50	.667	3.00
D3	1.00	2.00	1.00	5.00	1.000	5.00

Figure 2. Specimen with Hole and Gage Installation

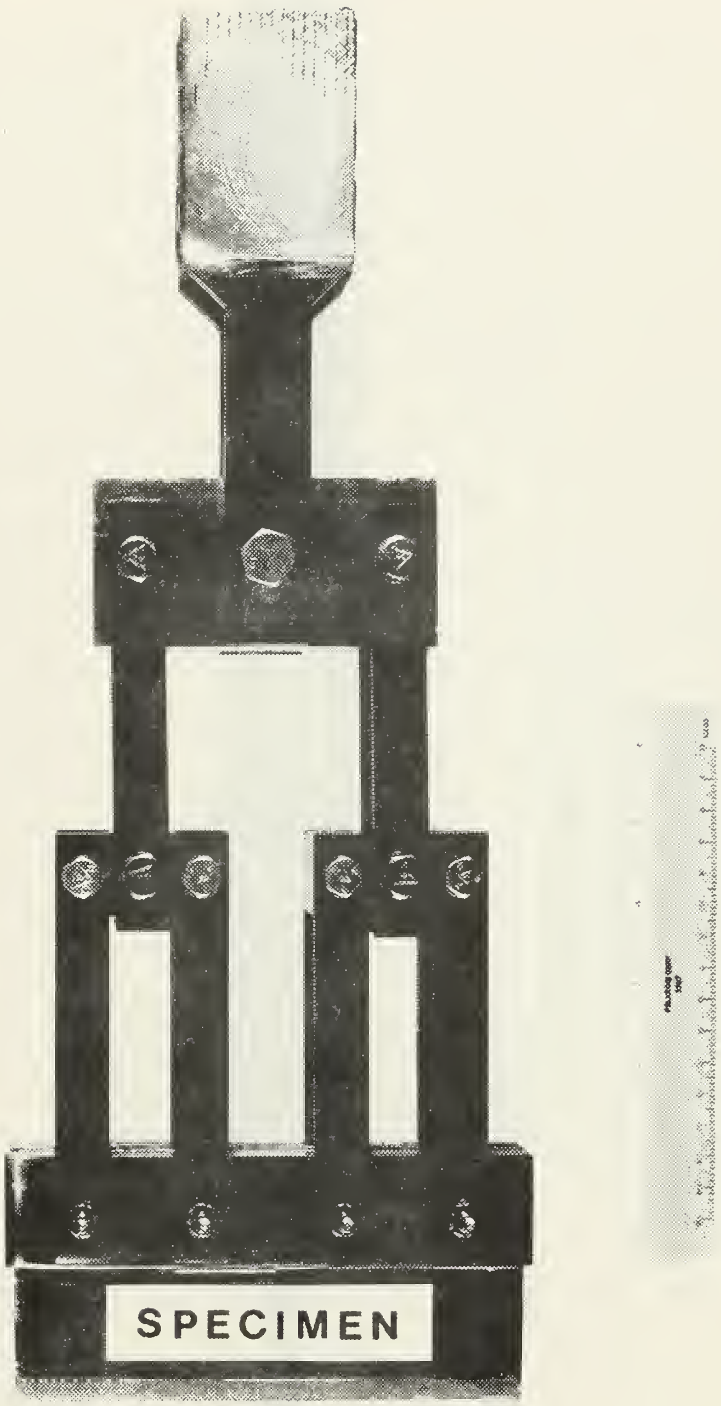

Figure 3-A. Whiffle Tree Arrangement (Front View)

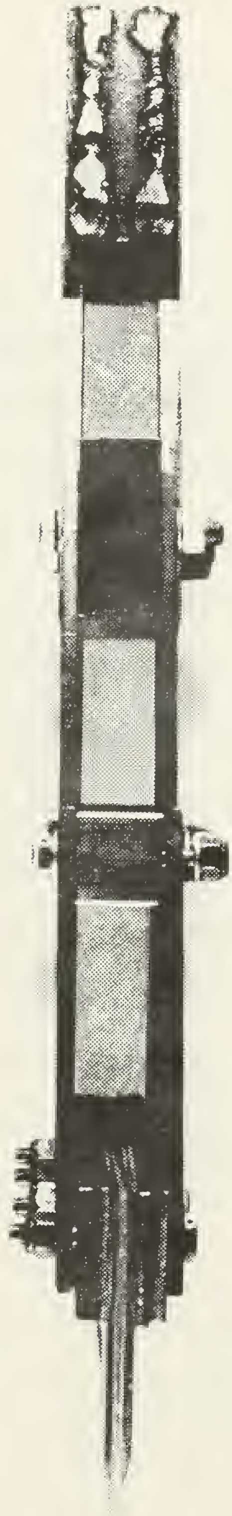

Figure 3-B. Whiffle Tree Arrangement (Side View)

V. MANUFACTURE OF THE SPECIMENS

The manufacture of the specimen was divided into two phases. The first phase was to make the machine to manufacture the specimens. The second phase was to manufacture the specimens.

The first phase, manufacture of machine to make specimens, was accomplished in three parts:

1. Manufacture of the heated platens (See Figures 4 and 5). There are two similar platens, one on the top and one on the bottom. Each platen was cut from aluminum with the dimension 38" x 26" x 3/4". Eight heater strips were installed on each aluminum plate. The heater strips were manufactured by Chromalox, with 1000 watts (74 watts per sq. in.), catalog number CIR-2120. After the heater strips were installed, 3/8" thick aluminum plates were attached to the platen, filling the gaps between heaters. Both heater strips and fillers were screwed down on the heater plates, with 1/2" space provided for the longitudinal expansion for each heater strip.
2. Manufacture of load-distributing structure (See Figures 6, 7, and 8). Six aluminum wide flange I-beams were cut and machined as flat as possible. Four of them are 38" long; two are placed on the top, and another two on the bottom. Both top and bottom beams were insulated

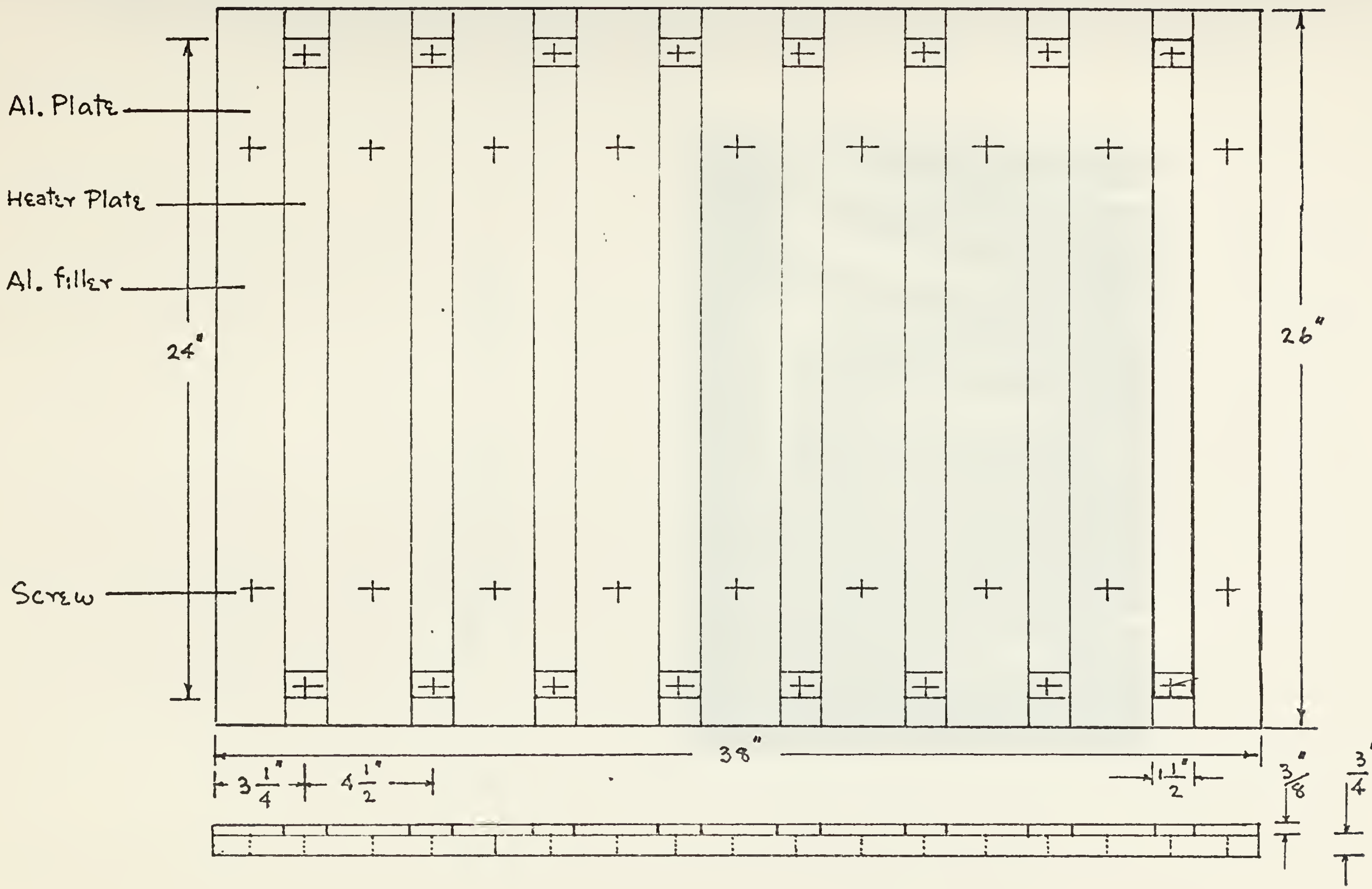

Figure 4. Heated Platen, Scale 6:1

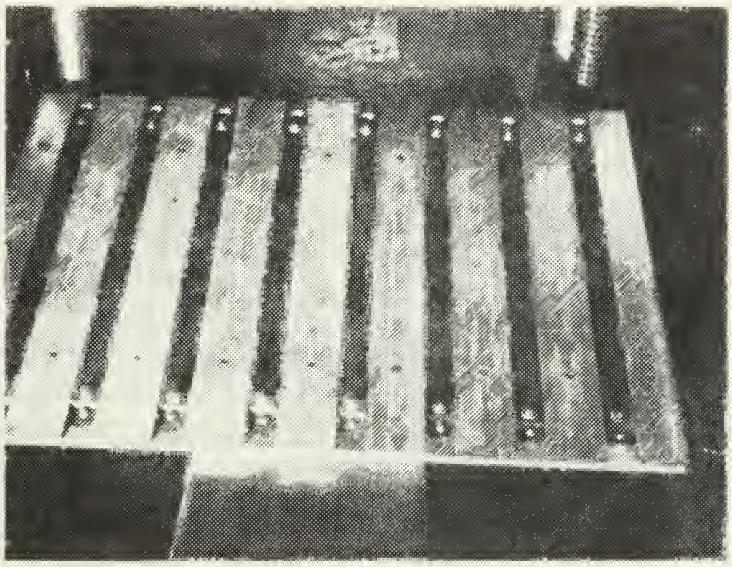

Figure 5. Heated Platens

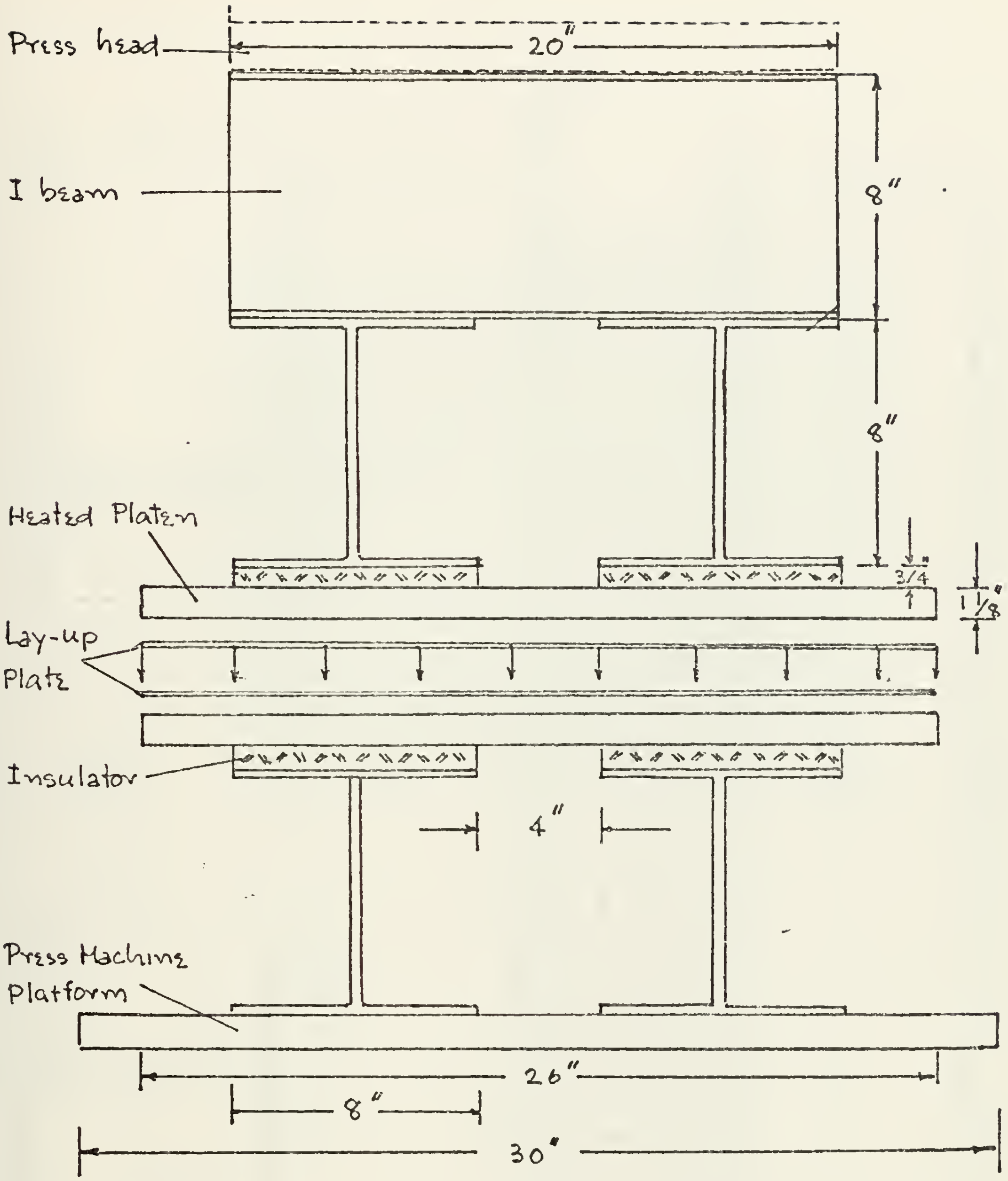

Figure 6. Press Arrangement Front View, Scale 6:1

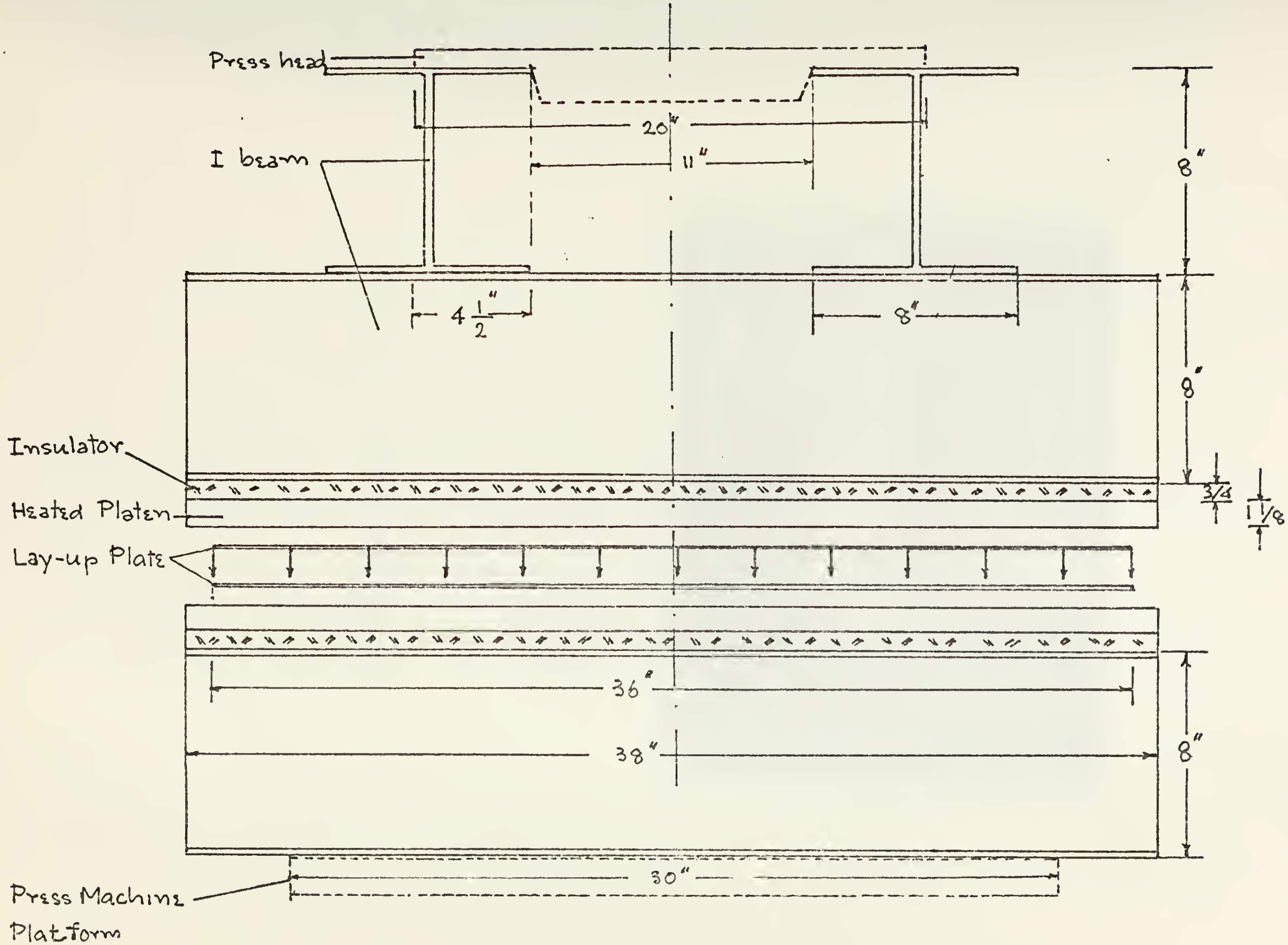

Figure 7. Press Arrangement Side View, Scale 6:1

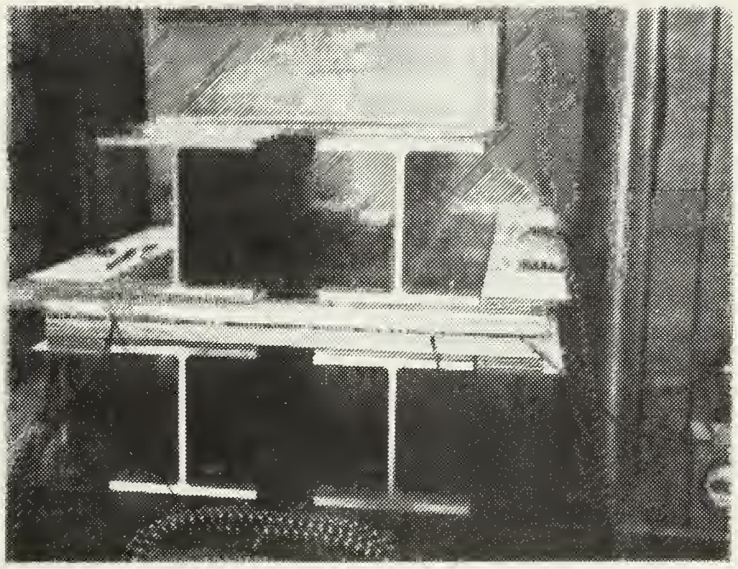

Figure 8. Press Arrangement

from the heater by 3/4" thick asbestos plates. Another two I-beams of the same dimension but 26" long were placed upon and 90° across the 38" long I-beams. The purpose of the I-beams is to distribute pressure equally from the press machine to the specimen.

3. Manufacture of lay-up plate. Initially two 36" x 24" x 3/4" aluminum plates were used. After the manufacture of the parts described above was finished, all parts were installed on the press machine. It was found that the lay-up plates did not meet each other evenly. Then all parts, including the I-beams with insulators, the heater strips from the heater plates and lay-up plates, were disassembled. All faces of the I-beams, heater plates and lay-out plates were machined down to make them very flat. But unfortunately during machine, the lay-up plates could not be made flat, since they bent due to residual stresses when machined. Finally the aluminum lay-up plates were replaced with stainless steel plates, 1/32 in. thick.

4. Instrumentation and control (See Figure 9). Thermal control during the lamination process was achieved using a Leeds and Northrup Speedomax-H Strip Chart Recorder and Series 60 Control Unit as the primary thermal control device. A thermocouple embedded in the laminate gave a temperature readout, and the Speedomax Controller activated the strip heater power

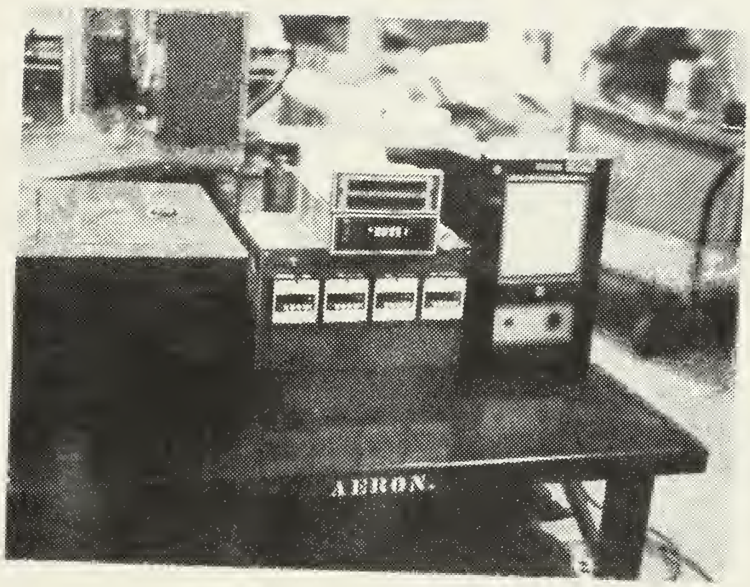

Figure 9. Instrumentation and Control

relay. To allow adequate time for even heating of the lay-up, the temperature rise in the laminate was limited to 4 - 6°F per minute. To control this temperature ramp, a gear train and electric motor were installed in the Speedomax, which drives the temperature limit switches upscale at 5°F per minute. Four "tankard" timers, manufactured by Automatic Timing and Controls, Inc., were used to control the cure cycle. They started the ramp motor to initiate temperature rises, stopped it during dwells, and shut off the main power relay when the cure cycle was completed.

Pressure control was accomplished manually by operating the controls of the universal test machine which was used as a press.

Before manufacture of the specimens, the parts described in the first phase were assembled and a free run was made to check the temperature distribution over the lay-up platen. Five iron-constantin thermocouples were installed in the lay-up plates at the positions shown in Figure 10 and the temperature readings were recorded in Table 2. It was found that the temperature distributions were satisfactory.

Phase two was manufacture of the specimen. The laminate consisted of nine layers of 0.007" thickness glass-eopxy tape oriented in cross-ply directions (i.e., 0", 90", 0", etc.). The fabrication of the laminate consisted of a curing cycle

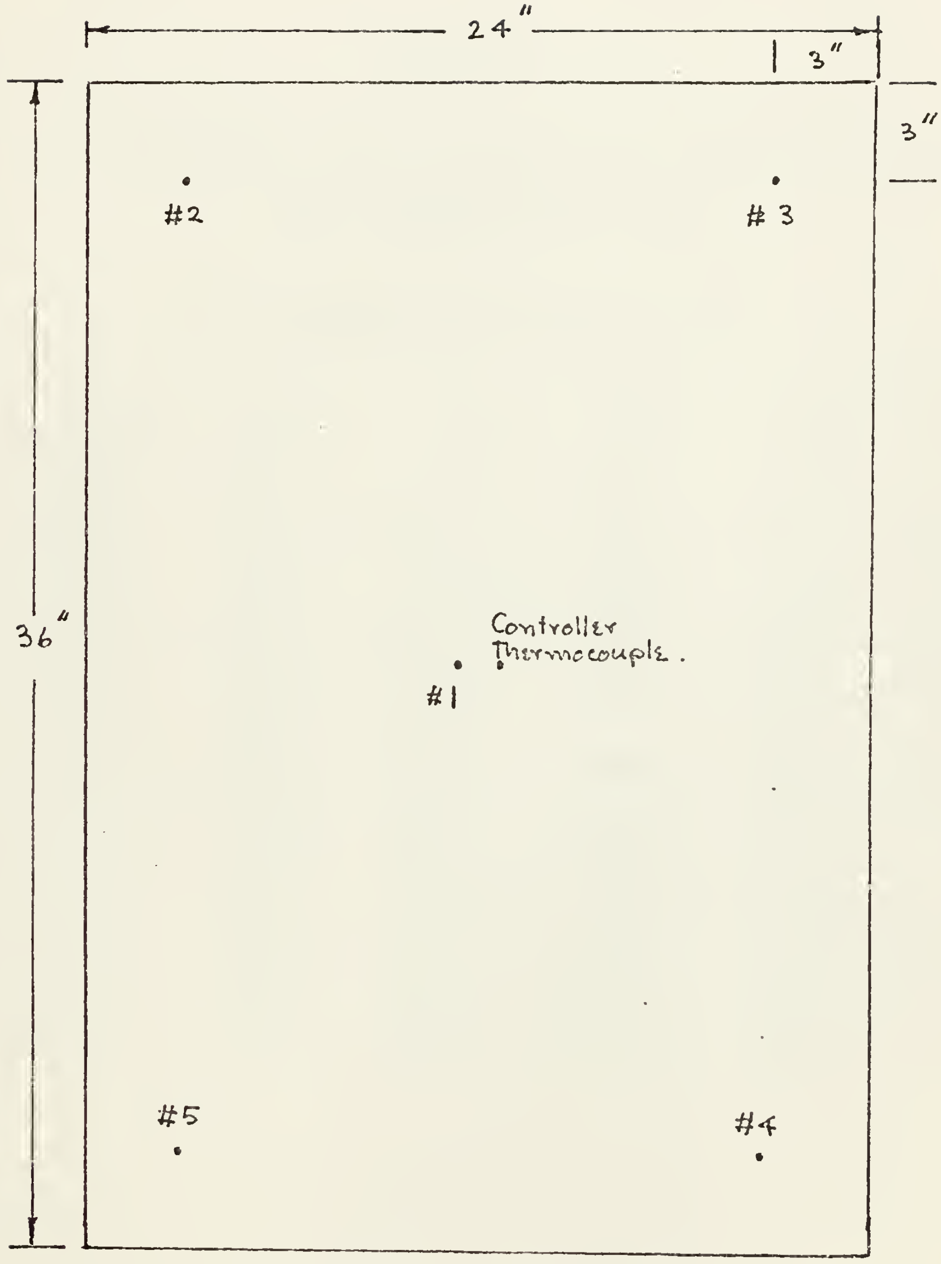

Figure 10. Position of Thermocouple
Installation in the Lay-up Plates

TABLE II

TEMPERATURE DISTRIBUTION IN THE LAY-UP PLATES

AS A FUNCTION OF TIME

Time	5. Thermocouple Reading (°F)					
(Min)	1	2	3	4	5	Controller
0	79.0	78.6	78.5	78.7	78.4	76.0
10	108.3	97.9	98.1	99.1	98.3	110.0
20	160.9	142.0	141.2	141.6	142.5	165.0
30	213.6	191.5	189.5	189.4	192.9	218.0
40	264.7	240.6	236.1	237.1	240.3	268.0
50	320.4	293.4	288.3	288.4	295.7	323.0
51.2	327.1	298.9	293.4	295.3	300.8	328.0
61.2	329.2	312.2	304.8	304.9	312.8	326.0
71.2	327.5	314.3	306.7	306.5	314.3	324.0
81.2	326.8	314.5	306.8	306.8	314.4	324.0

under pressure of 50 psi. The temperature was increased from the ambient temperature of 67°F to 330°F at a rate of 5.37°F/min, maintained at that temperature for 27 minutes, and then allowed to cool under pressure to the ambient temperature.

The finished laminate had dimensions of about 3' x 2' with 0.077" thickness, and was cut into four 6" x 36" specimens. Each one of these specimens represented a series corresponding to three different eccentricities, e, as shown in Table I.

Each of the specimens was prepared with grip tabs made of glass-epoxy, copper, and steel plate, as shown in Figure 2. The grip tabs were bonded to each of the specimen ends with epoxy cement.

A "whiffle-tree" linkage at each end of the specimen distributed the tensile load to the specimen end tabs (See Figures 3-A and 3-B). The end tabs then distributed the load to the specimen uniformly. The holes were cut, and after each test were enlarged, by means of a drill.

Two strain gages were installed inside the hole to measure strain at various loads. Another nine strain gages were installed across the specimen at a distance from the hole to measure the stress distribution pattern (See Figures 2 and 11). Gage installations were made by the Mechanical Engineering Laboratory shop, and the instructions recommended by the manufacturers were followed. The details of strain gages are described in Part VI.

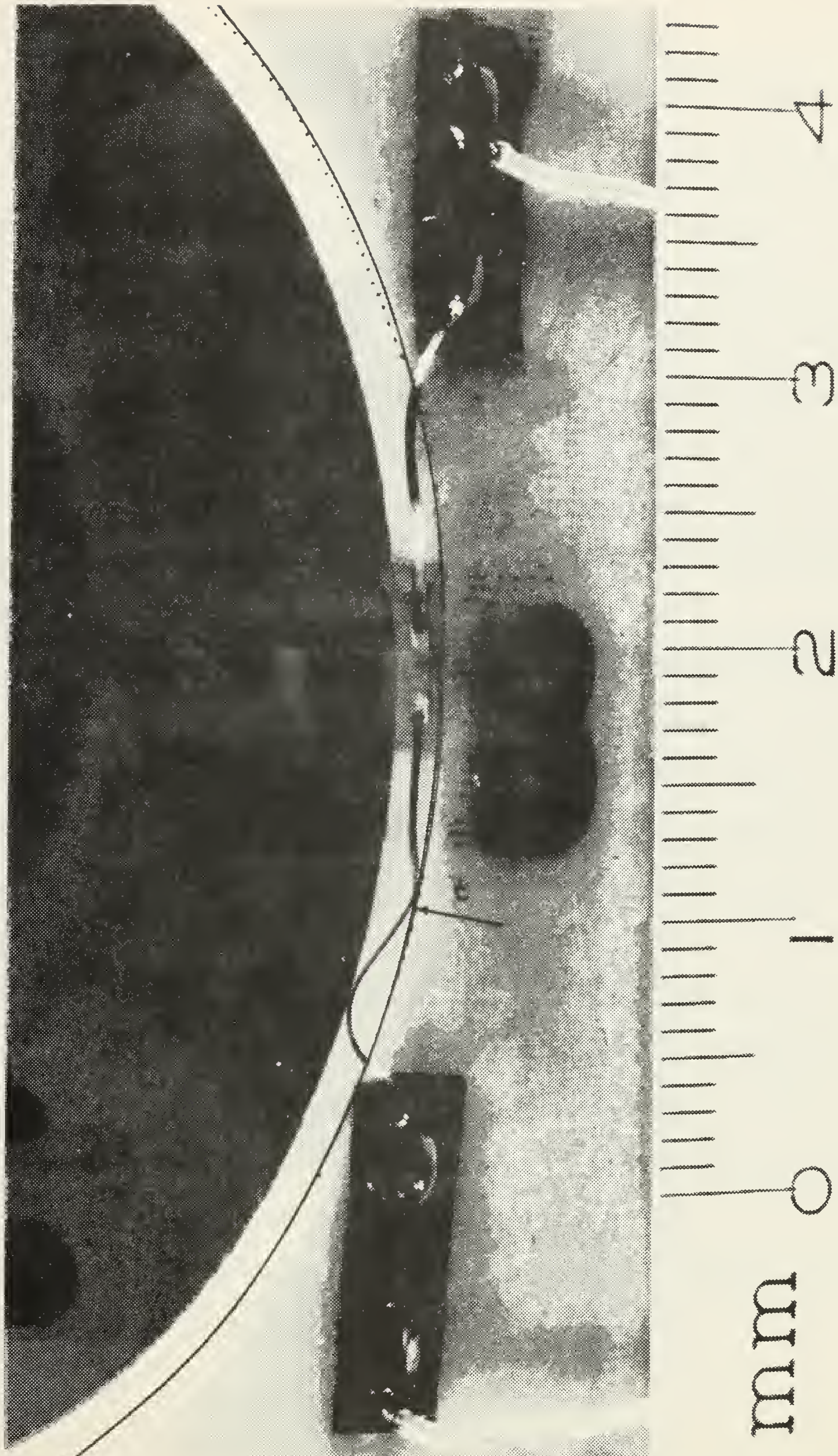

Figure 11. Installation of Small Gage inside the Hole

VI. INSTRUMENTATION AND TESTING APPARATUS

The tension test machine used was a Riehle Testing Machine with a maximum capacity of 60,000 pounds, located in the Aeronautics Laboratory. The machine was operated in its lowest range (0 - 15,000 lbs, 25 lbs per scale division), with the loading process controlled manually.

The gages were read with a Baldwin SR-4 Type N portable strain indicator, belonging to the Aeronautics Laboratory, which was used together with a Baldwin Switching and Balancing Unit.

The gages used were of the types listed below:

Type EA-06-031 DE-120, with the resistance of 120 ohms ±0.2% and gage factor of 2.02 ohms ±1.0%. These gages were manufactured by Micro-Measurements, Romulus, Michigan. They were used to measure the strain inside the hole.

Type C9-141, with the resistance of 350 ±.5%. These gages were manufactured by Budd Company, Phoenixville, Pennsylvania. They were used to measure the distribution of strain along the specimen's transverse direction at a significant distance from the hole.

The strain gage circuit used was the simple bridge arrangement with temperature compensation.

VII. EXPERIMENTAL PROCEDURE

Prior to beginning the main test series, two specimens, 0.5 x 3 x 0.77 in. in dimension, were cut from the manufactured plate to determine the maximum allowable load. It was found that these two specimens yielded and broke at about 2,000 lbs. load. The selected range of load for testing was decided to be between 0 and 1,250 lbs., with 125-lb. intervals.

The specimens were installed in the tension machine as shown in Figure 12. Eleven strain gages were attached to the specimen as shown in Figure 2. Gages A and B were used to measure the strain at the outer edge and inner edge of the hole, respectively, and gages numbered 1 through 9 were used to measure the stress-distribution across the plate in order to determine if the load was evenly distributed.

Testing started with specimen A-1 and continued through specimen D-3, twelve specimens in all. The series of experiments required approximately 120 hours. The data obtained is shown in Appendix A.

A final test was used to find the Young's modulus of the plate. Specimen E was tested in the region of 0 to 1,250 lbs., with 125-lb. increments. The results are shown in Appendix B and Figure 17.

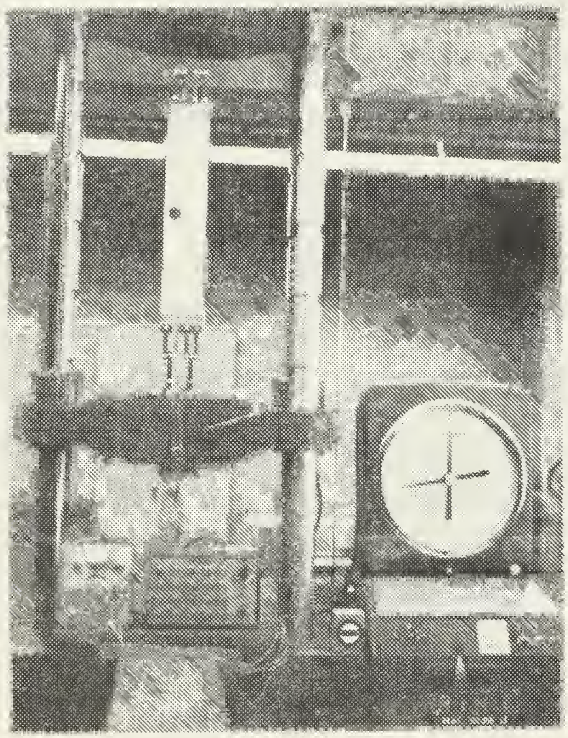

Figure 12. Installation of Specimen on the Tension Test Machine

VIII. PRESENTATION OF RESULTS

The data obtained in this test program may be found in Appendix A. The stress concentration factors computed from this data using Peterson's formula $K_{tg} = \frac{\sigma_H}{\sigma}$ [Ref. 3] are plotted in Figures 13 and 14 for K_{tg} at the outer edge and inner edge of the hole, respectively, as a function of the nondimensional parameters, a/c and e/c.

Parameter a/c represents the influence of the size of the hole, and e/c represents the influence of the eccentricity of the hole.

The stress concentration factor K_{tg} can be computed as follows:

$$K_{tg} = \frac{\sigma_H}{\sigma} = \frac{E\varepsilon}{\frac{P}{Ag}} = \frac{\varepsilon}{P} \cdot EAg$$

$$= 1.933488 \times 10^6 \frac{\varepsilon}{P}$$

The curves in Figures 13 and 14 show that at a/c = 0 (corresponding to plates of finite width) the value of K_{tg} is approximately 4.2.

In both Figures 13 and 14, the curve of e/c = 2. At the point a/c = .25 the value of K_{tg} = 4.467 (Figure 13) and 3.827 (Figure 14). This value seems to be too low; and by carefully comparing with the other results, it appears

that K_{tg} should fall between 4.695 and 5.180 for Figure 13 and between 4.175 and 4.970 for Figure 14. It is suspected that there was some error during data taking. Unfortunately this was not discovered until after the specimen had been modified for the next experiment. The data point is included but lightly weighted in drawing the curves.

TABLE III-A

EXPERIMENTAL RESULTS

Specimen	Gage A $\frac{\varepsilon}{p} \times 10^{-6}$	Gage B $\frac{\varepsilon}{p} \times 10^{-6}$	K_{tg} @ A	K_{tg} @ B
A-1	2.429	2.16	4.695 ± .468	4.175 ± .434
A-2	2.489	2.233	4.811 ± .478	4.316 ± .423
A-3	2.6	2.410	5.026 ± .508	4.659 ± .460
B-1	2.311	1.98	4.467 ± .437	3.827 ± .372
B-2	2.776	2.458	5.366 ± .521	4.751 ± .463
B-3	2.298	1.56	6.700 ± .784	5.49 ± .533
C-1	2.68	2.571	5.180 ± .503	4.97 ± .483
C-2	3.056	2.64	5.907 ± .585	5.103 ± .496
C-3	4.537	3.022	8.770 ± .860	5.842 ± .570
D-1	3.143	2.676	6.075 ± .590	5.173 ± .512
D-2	3.565	2.739	6.891 ± .672	5.294 ± .516
D-3	∞	5.4	______	10.438 ± 1.012

A summary of these results is shown in Table III-B

TABLE III-B

SUMMARY OF EXPERIMENTAL RESULTS

Specimen	e/c	a/c	K_{tg} @ A	K_{tg} @ B
A-1	2	.125	4.695 ± .468	4.175 ± .434
B-1	2	.25	4.467 ± .437	3.827 ± .372
C-1	2	.375	5.180 ± .503	4.97 ± .483
D-1	2	.500	6.075 ± .590	5.173 ± .512
A-2	3	.167	4.811 ± .478	4.316 ± .423
B-2	3	.333	5.366 ± .521	4.751 ± .463
C-2	3	.500	5.907 ± .585	5.103 ± .496
D-2	3	.667	6.891 ± .672	5.294 ± .516
A-3	5	.250	5.026 ± .508	4.659 ± .460
B-3	5	.500	6.700 ± .784	5.49 ± .533
C-3	5	.750	8.770 ± .860	5.842 ± .570
D-3	5	1.000	______	10.438 ± 1.012

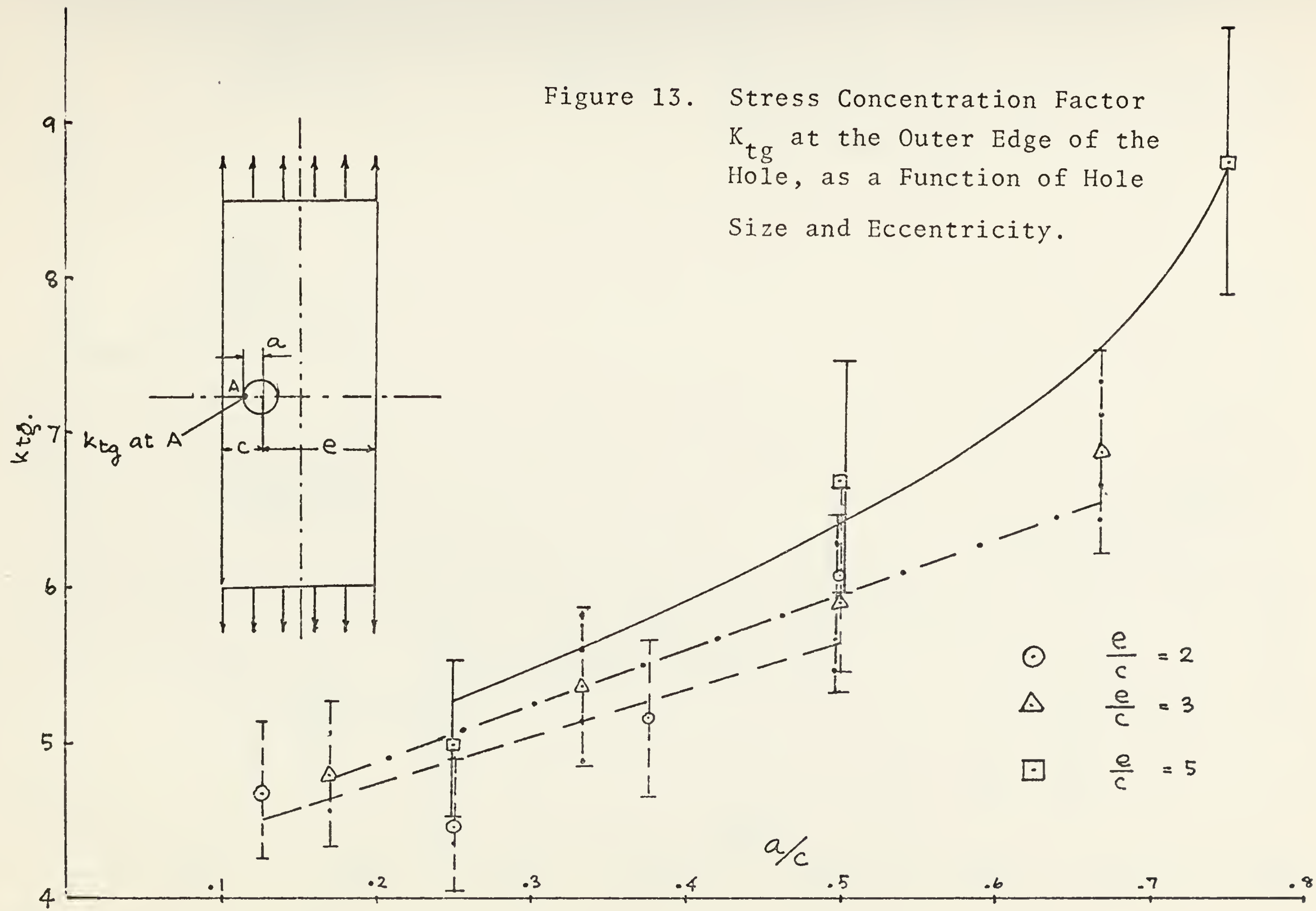

Figure 13. Stress Concentration Factor K_{tg} at the Outer Edge of the Hole, as a Function of Hole Size and Eccentricity.

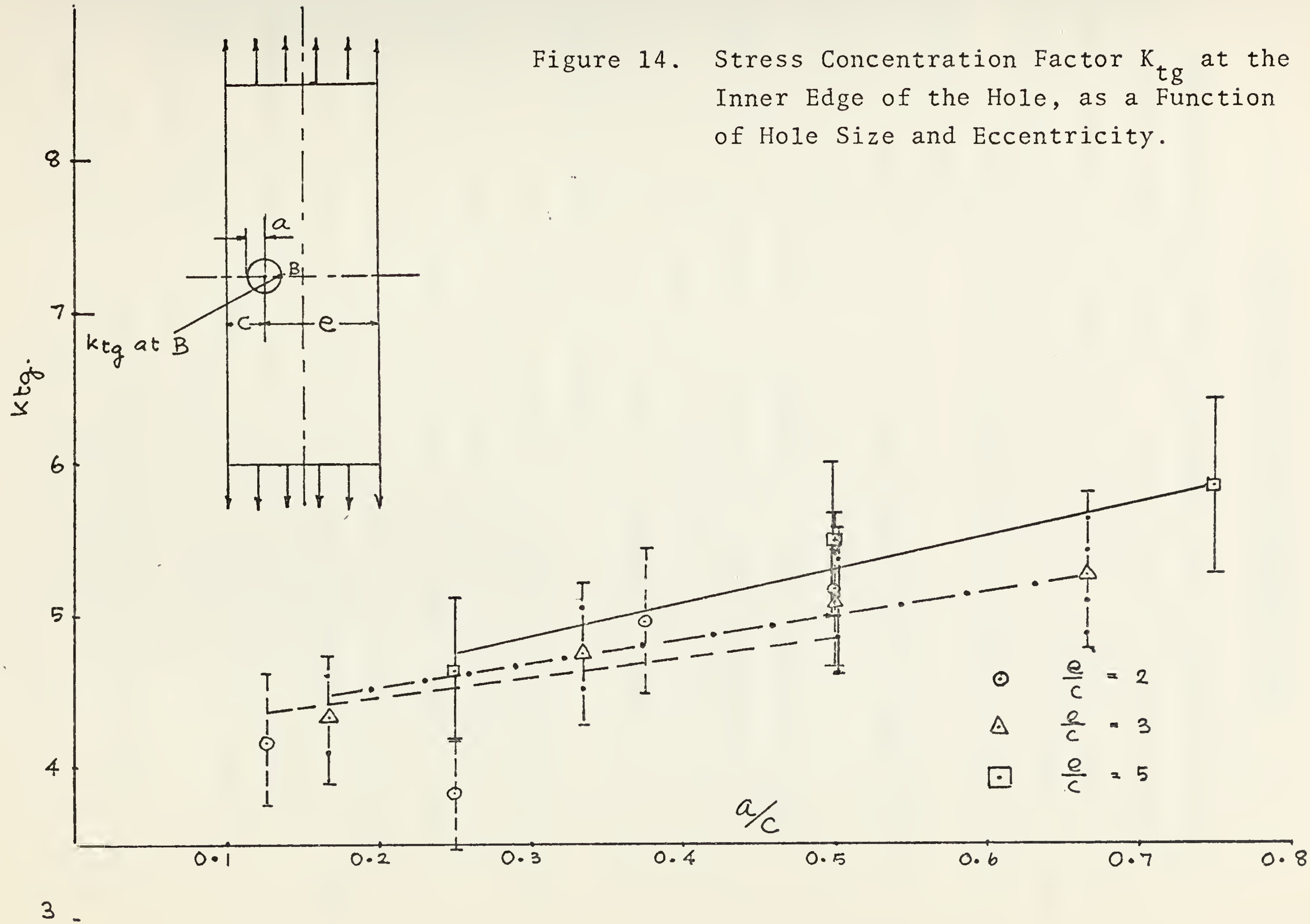

Figure 14. Stress Concentration Factor K_{tg} at the Inner Edge of the Hole, as a Function of Hole Size and Eccentricity.

IX. CONCLUSIONS AND RECOMMENDATIONS

The conclusions and recommendations presented here can be divided into two parts: (1) Conclusions and recommendations based on the experimental results, and (2) conclusions and recommendations based on experience gained during testing.

1. Conclusions and recommendations based on the experimental results.

The goals that were expected with the experimental program of this thesis were met because:

a. As shown in Appendix A, the Strain vs. Load curve for each of the 12 tests was linear (Figure 15).

b. The transverse distribution of strain away from the hole during the tests was essentially uniform, as can be seen from Figure 16, Appendix A.

c. As shown in Figures 13 and 14, the variations of K_{tg}, as a function of the dimensionless variables a/c and e/c, are in reasonable agreement with the previous results of Commander Alves [Ref. 5], with a/c = 0 (infinite plate). K_{tg} at the outer and inner curve was approximately 4.2. This agrees with Alves' result for unidirectional specimens (K_{tg} = 4.2) but is higher than his results for cross-ply specimens (K_{tg} = 3.5).

d. As the hole size increases, K_{tg} increases. As the eccentricity increases, K_{tg} increases. K_{tg} at the side

of the hole nearer to the edge of the specimen (point A) is greater than that at the other side (point B).

e. The effect of hole size on K_{tg} is greater when the eccentricity is increased, i.e., the slope of the K_{tg} vs. a/c line increases.

2. Conclusions and recommendations based on experience gained during testing.

a. The first problem of this experiment was the manufacture of the machine to make the specimen. It was difficult to achieve the required flatness of both upper and lower lay-out plates and also the heater plates. For future experiments it is recommended that plates be as flat and parallel as possible (See Figures 6, 7, and 8), in order to get equal thickness specimens.

b. The second problem was the difficulty of installing strain gages inside the hole, due to the very thin specimens. It is recommended that the specimen thickness used should be the minimum thickness allowable, in order to get a good strain gage bond in the hole.

c. Design of future specimens should be made with reference to Figures 13 and 14 so as to fill in the gaps in data for various values of a/c and e/c. This will allow more precise curves to be drawn.

d. It is also recommended that before enlarging the holes on the specimens, the plots of load vs. strain curves should be plotted and compared with each other.

If the data acquired appears incorrect, then the test can be performed once more for confirmation.

APPENDIX A

TEST DATA AND SAMPLE OF GAGE RESPONSE

In all of the cases, the behavior of the gages was similar to that shown in Figures 15 and 16.

TABLE IV. RESULT OF TESTS

Strain Gage Reading

Specimen A-1

Hole Radius (a) = 0.25 inch

Hole Center Location (f) = 1.00 inch

Hole Location Parameter c = 2.00 inches, e = 4.00 inches

Load	Average Strain (Microstrains)											
(lbs.)	Gage A	Gage B	Gage 1	Gage 2	Gage 3	Gage 4	Gage 5	Gage 6	Gage 7	Gage 8	Gage 9	Note
0												
125	310	280	100	100	110	100	100	85	95	110	100	
250	610	610	160	160	190	180	180	160	170	200	170	
375	880	840	250	250	275	270	270	235	270	275	250	
500	1185	1110	300	320	350	340	340	300	340	380	360	
625	1530	1375	390	400	440	415	425	360	430	450	430	
750	1810	1630	440	460	500	490	500	435	485	530	485	
875	2120	1865	515	540	620	575	585	520	575	660	605	
1000	2450	2120	560	600	665	650	650	590	635	670	635	
1125	2760	2400	630	670	725	710	725	625	710	730	700	
1250	3110	2670	690	735	790	780	790	680	770	800	780	

TABLE IV. RESULT OF TESTS

Strain Gage Readings

Specimen A-2

Hole Radius (a) = 0.25 inch

Hole Center Location (f) = 1.50 inches

Hole Location Parameter c = 1.50 inches, e = 4.50 inches

Load	Average Strain (Microstrains)											
(lbs.)	Gage A	Gage B	Gage 1	Gage 2	Gage 3	Gage 4	Gage 5	Gage 6	Gage 7	Gage 8	Gage 9	Note
0												
125	330	300	90	90	90	90	90	90	90	90	90	
250	630	615	155	145	155	155	155	150	165	165	165	
375	935	880	225	215	225	230	235	230	235	235	235	
500	1235	1155	290	280	290	295	300	295	300	300	300	
625	1540	1450	360	350	360	370	380	370	370	370	370	
750	1845	1670	430	410	430	440	460	430	440	440	440	
875	2180	1960	500	480	500	515	530	500	515	515	515	
1000	2490	2275	565	540	560	580	590	560	580	570	570	
1125	2800	2560	640	615	630	650	670	630	650	640	640	
1250	3125	2840	710	685	710	725	745	695	720	710	710	

TABLE IV. RESULT OF TESTS

Strain Gage Reading

Specimen A-3

Hole Radius (a) = 0.25 inch

Hole Center Location (f) = 2.00 inches

Hole Location Parameter c = 1.00 inches, e = 5.00 inches

Load (lbs.)	Average Strain (Microstrains) Gage A	Gage B	Gage 1	Gage 2	Gage 3	Gage 4	Gage 5	Gage 6	Gage 7	Gage 8	Gage 9	Note
0												
125	360	270	95	95	100	120	120	125	120	80	110	
250	660	570	140	140	150	160	160	160	150	100	160	
375	950	850	200	200	215	220	220	220	220	150	220	
500	1280	1175	270	270	290	290	295	290	290	190	300	
625	1620	1500	345	350	360	370	370	370	375	240	375	
750	1925	1795	410	410	420	440	440	430	430	280	445	
875	2240	2090	475	475	490	505	505	500	500	320	505	
1000	2570	2420	540	550	580	590	590	570	570	370	580	
1125	2920	2750	610	620	650	650	655	640	640	415	660	
1250	3250	3070	680	690	705	720	725	710	710	480	720	

TABLE IV. RESULT OF TESTS

Strain Gage Reading

Specimen B-1

Hole Radius (a) = 0.50 inch

Hole Center Location (f) = 1.00 inch

Hole Location Parameter c = 2.00 inches, e = 4.00 inches

Load (lbs.)	Average Strain (Microstrains) Gage A	Gage B	Gage 1	Gage 2	Gage 3	Gage 4	Gage 5	Gage 6	Gage 7	Gage 8	Gage 9	Note
0												Gage #6 should be changed.
125	290	260	70	90	70	80	90	60	100	80	80	
250	560	475	130	145	140	150	150	120	150	150	160	
375	810	720	205	230	220	230	245	175	240	235	235	
500	1120	960	270	295	295	285	315	245	310	290	310	
625	1430	1220	330	370	350	365	390	290	380	370	380	
750	1725	1480	400	430	430	450	460	370	460	440	460	
875	2045	1765	460	500	505	520	540	425	545	525	540	
1000	2335	2000	530	580	580	575	620	500	610	590	610	
1125	2655	2280	580	640	640	660	690	540	685	665	690	
1250	2960	2560	650	710	720	710	760	610	740	730	740	

TABLE IV. RESULT OF TESTS

Strain Gage Reading

Specimen B-2

Hole Radius (a) = 0.50 inch

Hole Center Location (f) = 1.50 inches

Hole Location Parameter c = 1.50 inches, e = 4.50 inches

Load (lbs.)	Average Strain (Microstrains) Gage A	Gage B	Gage 1	Gage 2	Gage 3	Gage 4	Gage 5	Gage 6	Gage 7	Gage 8	Gage 9	Note
0												
125	300	280	55	55	55	65	70	70	65	65	65	
250	670	580	130	130	130	130	140	140	130	130	130	
375	1045	915	200	200	200	210	215	215	200	200	200	
500	1375	1200	265	260	260	270	280	275	270	270	270	
625	1730	1520	360	360	360	365	365	370	375	375	375	
750	2055	1810	420	410	430	420	440	440	440	460	470	
875	2440	2155	520	500	510	510	520	520	530	520	525	
1000	2800	2475	580	575	575	580	600	600	600	600	600	
1125	3185	2820	620	610	620	630	640	645	635	630	640	
1250	3530	3120	685	670	685	690	715	710	710	700	715	

TABLE IV. RESULT OF TESTS

Strain Gage Reading

Specimen B-3

Hole Radius (a) = 0.5 inch

Hole Center Location (f) = 2.00 inches

Hole Location Parameter c = 1.00 inches, e = 5.00 inches

Load	Average Strain (Microstrains)											
(lbs.)	Gage A	Gage B	Gage 1	Gage 2	Gage 3	Gage 4	Gage 5	Gage 6	Gage 7	Gage 8	Gage 9	Note
0												Gage #8 is broken
125	450	400	60	60	75	65	60	50	60	40	60	
250	875	750	130	130	140	130	130	120	130	90	130	
375	1300	1050	190	190	200	220	210	190	210	150	220	
500	1725	1400	285	280	290	300	290	270	280	200	290	
625	2175	1750	360	350	390	440	430	410	430	310	445	
750	2575	2100	470	480	490	500	500	480	500	360	515	
875	3050	2500	540	540	570	560	555	540	560	400	570	
1000	3450	2825	580	590	610	630	630	600	630	430	630	
1125	3975	3175	665	670	700	710	690	670	690	460	670	
1250	4330	3525	680	690	710	720	710	690	710	490	740	

TABLE IV. RESULT OF TESTS

Strain Gage Reading

Specimen C-1

Hole Radius (a) = 0.75 inch

Hole Center Location (f) = 1.00 inch

Hole Location Parameter c = 2.00 inches, e = 4.00 inches

Load	Average Strain (Microstrains)											
(lbs.)	Gage A	Gage B	Gage 1	Gage 2	Gage 3	Gage 4	Gage 5	Gage 6	Gage 7	Gage 8	Gage 9	Note
0												
125	290	290	60	60	70	70	70	75	75	75	80	
250	640	640	130	145	150	150	150	160	160	160	160	
375	950	940	200	210	220	220	220	220	220	220	220	
500	1290	1265	260	280	280	290	295	300	290	300	305	
625	1625	1590	315	350	360	360	370	380	380	380	380	
750	1965	1915	380	415	420	420	440	445	445	440	450	
875	2350	2290	455	490	500	505	520	530	530	525	540	
1000	2695	2630	520	560	570	570	590	600	590	595	610	
1125	3080	3000	580	635	640	645	670	670	680	665	685	
1250	3400	3330	645	685	685	700	725	735	740	720	755	

TABLE IV. RESULT OF TESTS

Strain Gage Reading

Specimen C-2

Hole Radius (a) = 0.75 inch

Hole Center Location (f) = 1.50 inches

Hole Location Parameter c = 1.50 inches, e = 4.50 inches

Load	Average Strain (Microstrains)										
(lbs.)	Gage A	Gage B	Gage 1	Gage 2	Gage 3	Gage 4	Gage 5	Gage 6	Gage 7	Gage 8	Gage 9	Note
0												
125	360	310	70	70	60	70	70	70	70	70	70	
250	775	670	145	130	130	140	150	140	145	145	145	
375	1145	990	200	180	200	200	205	205	200	200	200	
500	1530	1325	270	250	260	270	280	280	280	275	275	
625	1920	1650	340	305	325	340	360	345	350	345	345	
750	2290	1970	410	365	390	400	410	410	410	400	405	
875	2720	2340	490	440	470	475	490	500	490	480	480	
1000	3100	2670	555	500	530	545	560	560	560	550	550	
1125	3525	3030	630	570	610	620	635	635	635	620	630	
1250	3900	3340	695	630	670	680	700	700	700	685	690	

TABLE IV. RESULT OF TESTS

Strain Gage Reading

Specimen C-3

Hole Radius (a) = 0.75 inch

Hole Center Location (f) = 2.00 inches

Hole Location Parameter c = 1.00 inches, e = 5.00 inches

Load	Average Strain (Microstrains)											
(lbs.)	Gage A	Gage B	Gage 1	Gage 2	Gage 3	Gage 4	Gage 5	Gage 6	Gage 7	Gage 8	Gage 9	Note
0												
125	640	410	60	70	70	80	80	70	70	60	60	
250	1160	760	120	130	140	140	140	140	135	130	130	
375	1720	1135	190	190	205	210	220	200	200	190	200	
500	2290	1520	250	260	280	280	280	260	260	260	265	
625	2860	1900	320	330	350	360	360	330	335	330	335	
750	3410	2270	385	400	420	425	430	390	400	390	405	
875	4030	2670	460	475	500	510	515	480	480	470	490	
1000	4510	3000	515	530	560	570	570	540	540	530	540	
1125	5110	3390	585	600	630	640	650	600	610	600	615	
1250	5670	3760	650	670	700	710	720	670	675	665	680	

TABLE IV. RESULT OF TESTS

Strain Gage Reading

Specimen D-1

Hole Radius (a) - 1.00 inch

Hole Center Location (f) = 1.00 inch

Hole Location Parameter c = 2.00 inches, e = 4.00 inches

Load	Average Strain (Microstrains)											
(lbs.)	Gage	Gage	Gage	Gage	Gage	Gage	Gage	Gage	Gage	Gage	Gage	Note
0												
125	415	330	80	80	70	70	80	90	80	85	90	
250	810	670	145	150	140	140	155	160	155	155	150	
375	1165	970	200	210	200	200	215	215	215	215	225	
500	1530	1300	265	280	300	280	300	290	320	290	290	
625	1955	1670	335	350	340	345	370	370	370	370	380	
750	2330	2000	395	420	405	410	435	445	440	445	455	
875	2740	2340	460	485	470	470	500	510	505	500	520	
1000	3120	2660	520	555	535	540	575	580	580	575	590	
1125	3565	3035	600	630	610	620	650	660	660	655	675	
1250	3995	3400	665	700	680	690	720	730	730	730	745	

TABLE IV. RESULT OF TESTS

Strain Gage Reading

Specimen D-2

Hole Radius (a) = 1.00 inch

Hole Center Location (f) = 1.50 inches

Hole Location Parameter c = 1.50 inches, e = 4.50 inches

Load	Average Strain (Microstrains)											
(lbs.)	Gage A	Gage B	Gage 1	Gage 2	Gage 3	Gage 4	Gage 5	Gage 6	Gage 7	Gage 8	Gage 9	Note
0												
125	515	360	60	70	65	75	100	75	80	70	80	
250	920	680	110	120	120	120	145	130	130	130	130	
375	1340	1015	180	180	180	195	220	200	200	200	200	
500	1715	1315	255	255	255	255	290	270	270	270	270	
625	2235	1720	315	315	315	325	355	335	335	335	335	
750	2680	2080	390	390	390	390	430	410	410	410	410	
875	3120	2400	450	450	450	460	490	480	480	470	470	
1000	3525	2730	520	520	520	520	565	550	550	540	550	
1125	4040	3110	590	590	590	590	630	620	620	600	615	
1250	4510	3490	655	655	655	660	710	690	695	680	690	

TABLE IV. RESULT OF TESTS

Strain Gage Reading

Specimen D-3

Hole Radius (a) = 1.00 inch

Hole Center Location (f) = 2.00 inches

Hole Location Parameter c = 1.00 inches, e = 5.00 inches

Load	Average Strain (Microstrains)											
(lbs.)	Gage A	Gage B	Gage 1	Gage 2	Gage 3	Gage 4	Gage 5	Gage 6	Gage 7	Gage 8	Gage 9	Note
0												
125		620	60	60	70	75	75	60	50	60	60	
250		1325	120	120	145	150	155	140	130	135	140	
375		2040	180	180	210	220	230	215	205	200	200	
500		2715	240	240	270	290	300	270	270	260	265	
625		3385	290	305	345	365	375	340	340	320	330	
750		4030	370	370	420	445	460	420	415	400	400	
875		4700	420	430	490	515	530	490	480	460	470	
1000		5340	460	480	555	590	610	555	550	520	530	
1125		6035	520	540	630	670	700	630	620	595	600	
1250		6770	595	620	710	765	790	720	710	670	685	

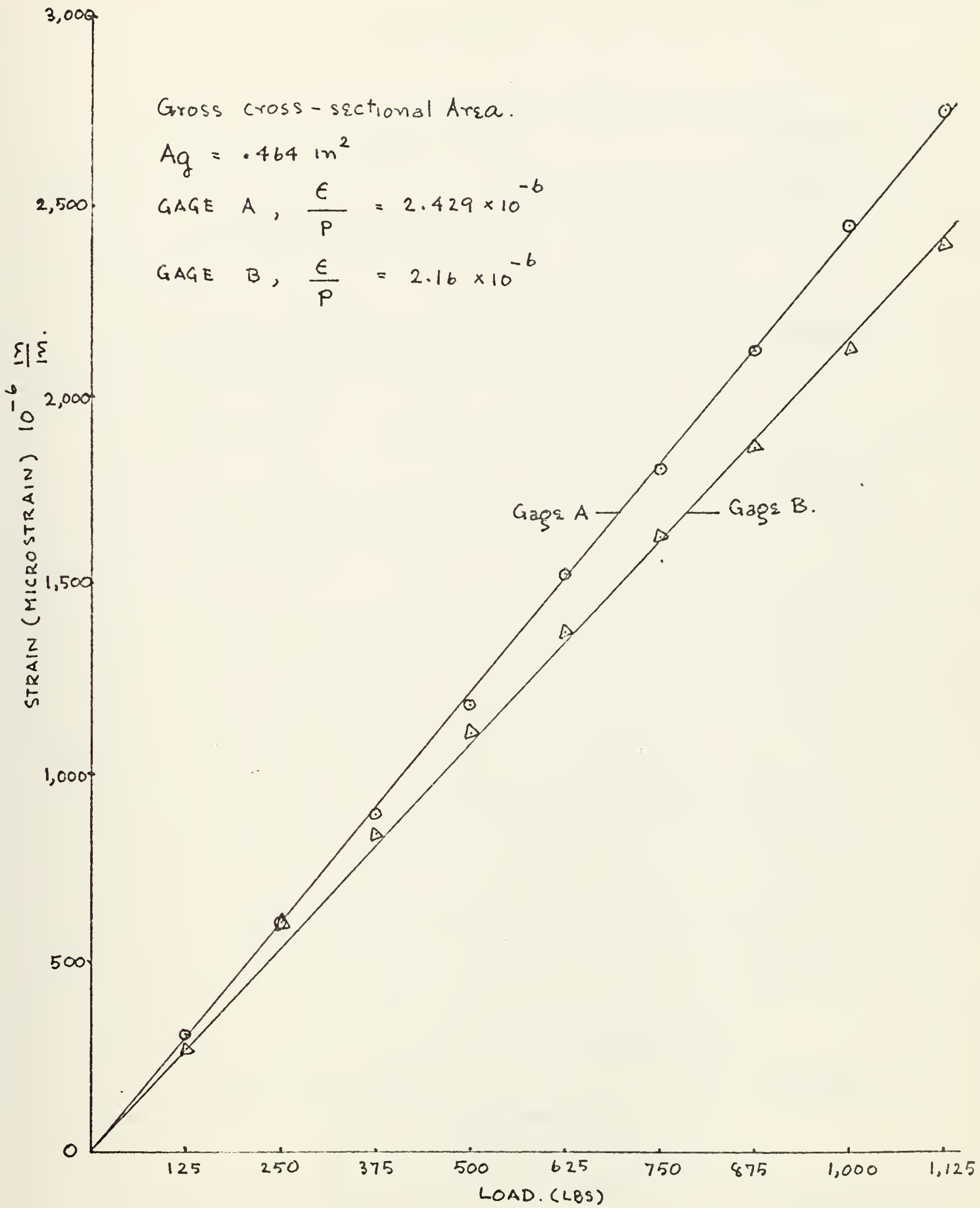

Figure 15. Strain at Sides of the Hole as a Function of Load

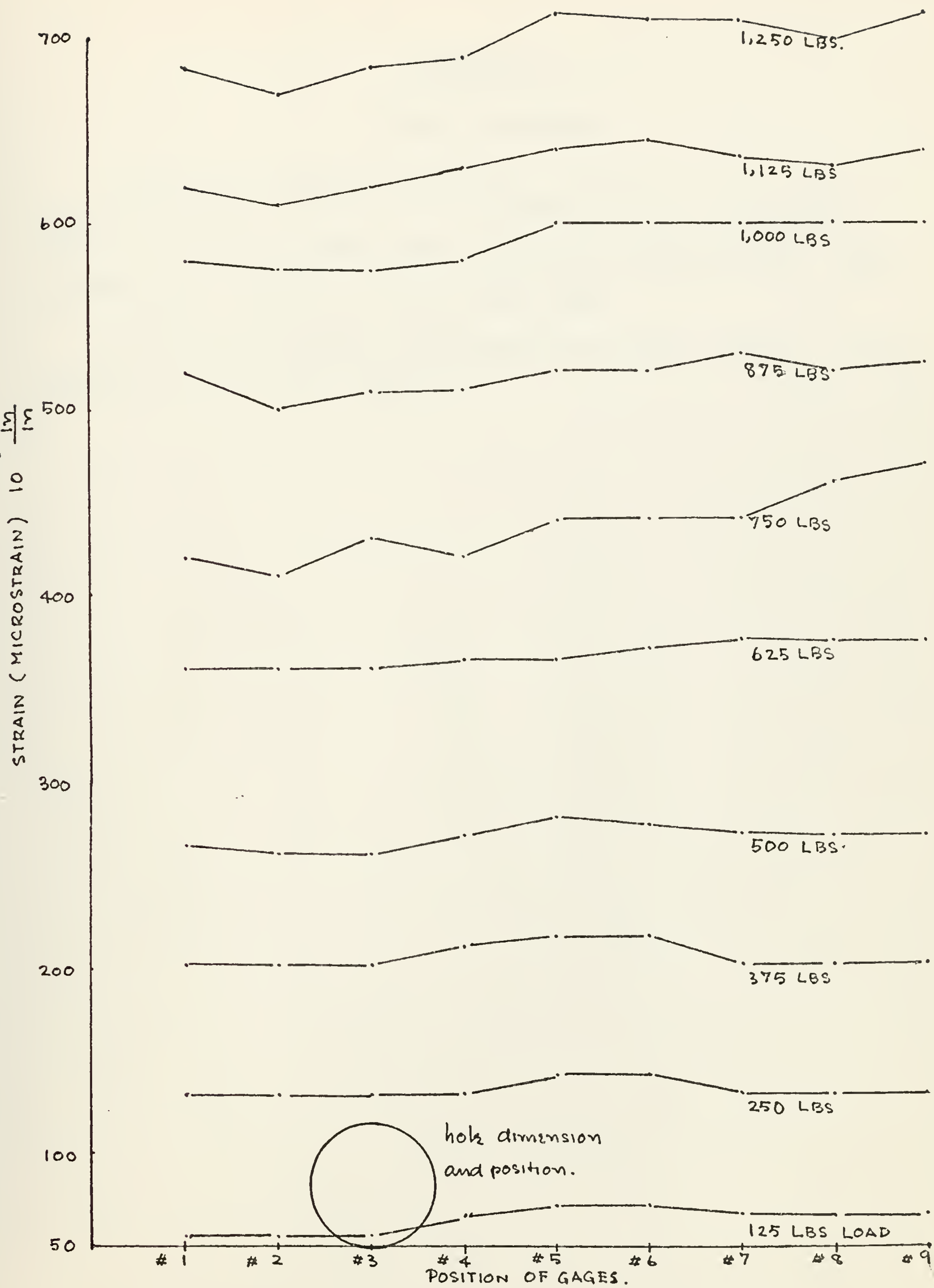

Figure 16. Transverse Distribution of Strain away from the Hole

APPENDIX B

YOUNG'S MODULUS

The Young's Modulus was obtained from a rectangular specimen with a strain gage installed at the center. The strain gage used was of the foil type.

The Young's Modulus obtained was 4.167×10^6 psi, with an error of $\pm .151 \times 10^6$ psi.

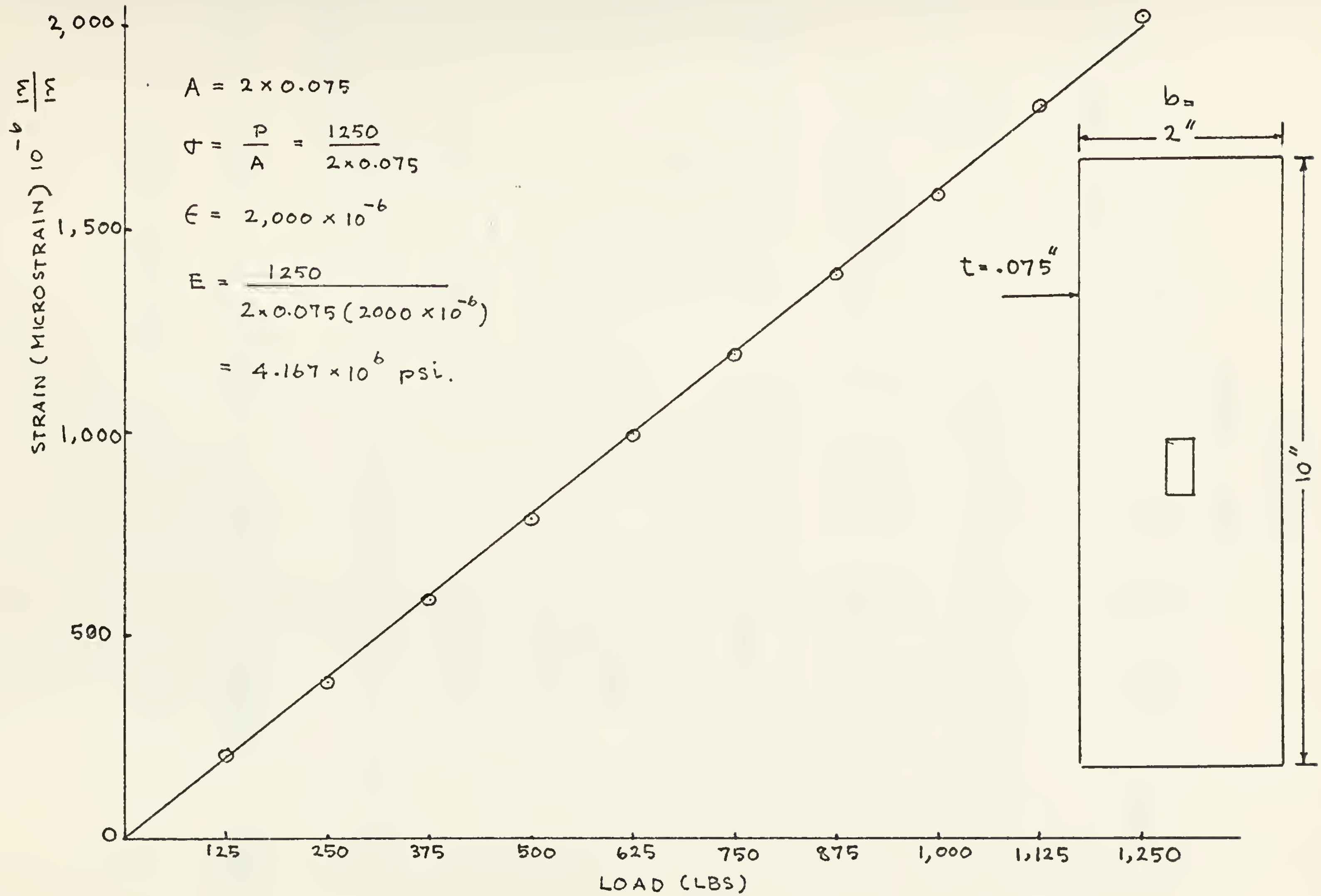

Figure 17. Strain as a Function of Load for E

APPENDIX C

ERROR ANALYSIS

The error analysis for Young's Modulus was based on the following equation [Ref. 4]:

$$E = \frac{s}{\varepsilon} = \frac{P}{b \times t \times \varepsilon} = 4.167 \times 10^6 \text{ psi} \quad \text{(See Figure 17)}$$

$$\omega E = \pm E[(\frac{\omega_P}{P})^2 + (\frac{\omega_b}{b})^2 + (\frac{\omega_t}{t})^2 + (\frac{\omega_\varepsilon}{\varepsilon})^2]^{1/2}$$

where the uncertainties involved in Young's Modulus are:

$\omega_P = \pm$ 25 lbs.	P = 1,250 lbs.
$\omega_\varepsilon = \pm$ 20 microstrain	ε = 2,000
$\omega_b = \pm$ 0.02 in.	b = 2.00 in.
$\omega_t = \pm$ 0.002 in.	t = 0.075 in.

The value of Young's Modulus (E) obtained has the relative error of

$$\omega_E = \pm\ 4.167 \times 10^6[(\frac{25}{1250})^2 + (\frac{0.02}{2})^2 + (\frac{0.002}{0.075})^2 + (\frac{20}{2000})^2]^{1/2}$$

$$= \pm\ .151 \times 10^6 \text{ psi}$$

The error analysis for the stress concentration factor K_{tg} was based on:

$$K_{tg} = \frac{\sigma_H}{\sigma_g} = \frac{E\varepsilon Ag}{P} = \frac{E \cdot \varepsilon \cdot b \cdot t}{P}$$

$$\omega_{K_{tg}} = \pm K_{tg}\left[\left(\frac{\omega_E}{E}\right)^2 + \left(\frac{\omega_\varepsilon}{\varepsilon}\right)^2 + \left(\frac{\omega_b}{b}\right)^2 + \left(\frac{\omega_t}{t}\right)^2 + \left(\frac{\omega_P}{P}\right)^2\right]^{1/2}$$

where the uncertainties involved in K_{tg} are

$\omega_E = \pm .151 \times 10^6$ psi $\qquad E = 4.167 \times 10^6$ psi

$\omega_\varepsilon = \pm 20$ microstrain

$\omega_b = \pm .05$ in. $\qquad b = 6$ in.

$\omega_t = \pm 0.007$ in. $\qquad t = 0.077$ in.

$\omega_P = \pm 25$ lbs.

The results of this analysis for each test are listed in Table III and displayed in Figures 13 and 14.

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